PROBLEM OF THE FLOW AND HYDRAULIC RESISTANCE OF A FLUID OF VARIABLE VISCOSITY

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In the chemical, petroleum refining, and food industries, and in medicine, fluids with structural viscosity, defined by the rheological curve shown in Fig. 1., are widely used. In [1] a method is proposed for describing the rheological properties of such media and the appropriateness is indicated of defining the class of these fluids by a linear fluidity law in the region of stresses close to τ_1.

In practice there can also occur flows of these fluids in the region of stresses close to τ_2, when a gradual transition to motion of the medium with the largest practical constant fluidity φ_2 is observed. Thus, it is to be expected that the flow of blood in the blood stream is similar in man and animals as the pressure and other pathological states are reduced [2].

1. Let τ_2 denote the value of the shear stress such that when $\tau > \tau_2$ the motion of the medium can be assumed to have constant fluidity φ_2. We approximate the part of the rheological curve near τ_2 by a logarithmic function, the inverse of the exponential relation proposed in [1]

$$\varphi_* = 0, \quad \tau \geqslant \tau_2, \quad \varphi_* = \ln \tau_*, \quad \tau \leqslant \tau_2 \tag{1.1}$$

where τ_* and φ_* are nondimensional variables

$$\tau_* = \frac{\tau - \tau_1}{\tau_2 - \tau_1}, \quad \varphi_* = \frac{1}{\theta} \frac{\varphi - \varphi_2}{\tau_2 - \tau_1}$$

(θ is a measure of the structural stability of the fluid), which satisfy (1.1) when $\tau = \tau_2$. The behavior of these variables as $\tau \to \tau_1$ can be ignored, since the point of view proposed here refers to the region of τ_* close to unity.

For structured fluids with linear fluidity law in the region of τ close to τ_2 we obtain a simple rheological equation $\varphi = \varphi_2 - \theta(\tau_2 - \tau)$, which contains variables defining the upper part of the flow curve under consideration.

2. Consider the laminar isothermal flow of the fluid being studied with structural viscosity in a circular cylindrical channel of radius R with rigid walls. Such a flow can be observed, for example, in the motion of the blood in vessels of constant aperture (sclerotic vessels).

Proceeding as in [1] we obtain, using (1.1), an equation for the velocity profile of the flow of a structured fluid

$$w = {}^1\!/_2 \varphi_2 R (\tau_c - \tau_c') \left[(1 - \xi^2) + {}^2\!/_3 \vartheta (\tau_c'' - \tau_c') \ln \frac{\tau_c - \tau_c'}{\tau_c'' - \tau_c'} (1 - \xi^3) \right]$$

and the mean stream velocity

$$\langle w \rangle = {}^1\!/_4 \varphi_2 R (\tau_c - \tau_c') \left[1 + {}^4\!/_5 \vartheta (\tau_c'' - \tau_c') \ln \frac{\tau_c - \tau_c'}{\tau_c'' - \tau_c'} \right]$$

where τ_c is the shear stress at the channel wall and ξ, ϑ, τ_c' and τ_c'' are defined by the equations

$$\xi = \frac{r}{R}, \quad \vartheta = \frac{\theta}{\varphi_2}, \quad \tau_1 = \tau_c' \xi, \quad \tau_2 = \tau_c'' \xi$$

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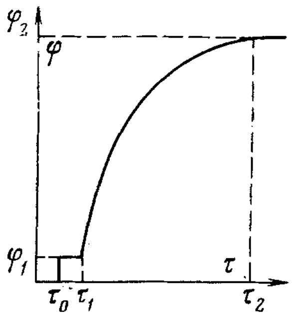

Fig. 1

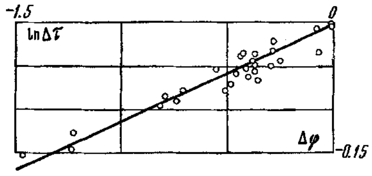

Fig. 2

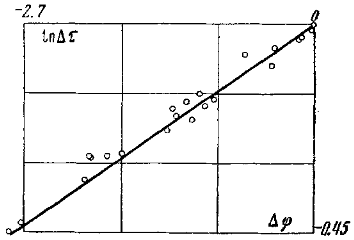

Fig. 3

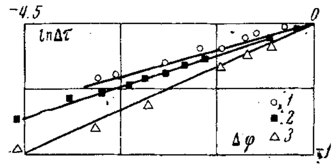

Fig. 4

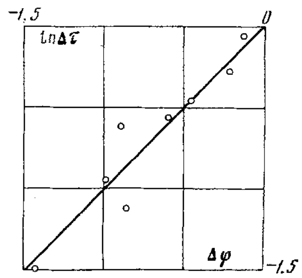

Fig. 5

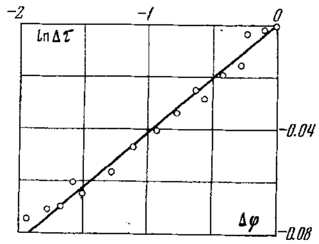

Fig. 6

Introducing nondimensional variables by analogy with φ_* and τ_*,

$$\Delta\varphi = \frac{\varphi_k - \varphi_2}{\tau_c'' - \tau_c'}, \qquad \Delta\tau = \frac{\tau_c - \tau_c'}{\tau_c'' - \tau_c'}$$

(φ_k is the apparent fluidity of the medium), on the basis of the last equation we reach the conclusion that $\Delta\varphi$ is a linear function of ln $\Delta\tau$. Figures 2 and 3 show the experimental results of the author from the investigation of the flow of blood in steel pipes. In these experiments the fluid flows through straight horizontal cylindrical pipes of various diameters (3-7 mm) and lengths under the action of the pressure due to the pressure tank in which the level is kept constant. By lateral spurs, manometers are connected to these pipes to measure the pressure difference between the ends of the section under investigation. The pressure drop is controlled by a tap beyond the experimental section at the open end of the system. The fluid flow rate per second was measured in the usual way. The experimental apparatus ensuring laminar flow with a fully developed velocity profile was tested with water.

The straight lines in Fig. 4 were obtained for blood of various concentrations from experimental results. Figures 5 and 6 show experimental results from the flows of other structured fluids: bitumen and a solution of rubber and toluene [1]. Figures 2-6 show that the experimental results (circles) lie on straight lines.

3. We introduce the following variables which define the hydraulic resistance of the motion of a fluid with structured viscosity:

$$\lambda = \frac{8(\tau_c - \tau_c')}{\rho\langle w\rangle^2}, \qquad \lambda_N = \frac{8(\tau_N - \tau_c')}{\rho\langle w\rangle^2}$$

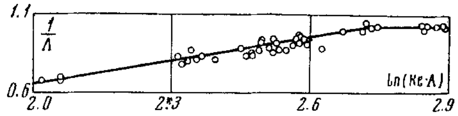

Fig. 7

where λ is the hydraulic resistance coefficient of the flow of a structured fluid at some average velocity $\langle \omega \rangle$, and λ_N is the hydraulic resistance coefficient for a Newtonian fluid of fluidity φ_2 at the same flow velocity; τ_N is the shear stress at the channel wall at which the average flow velocity of a Newtonian fluid with fluidity φ_2 is $\langle \omega \rangle$. We note that for the region of shear stress under consideration, when we can neglect τ_c', the above definitions of λ and λ_N coincide with the usual definitions.

We characterize the magnitude of the additional hydraulic losses due to the departure of the behavior of a fluid with structured viscosity from the Newtonian in $\tau < \tau_2$ by the relative resistance coefficient which is defined by the equation

$$\lambda = \Lambda \lambda_N$$

If the Reynolds number for a fluid with variable viscosity is defined as in [1], $\mathrm{Re} = 2\rho \langle \omega \rangle R\varphi_2$, we note that for given choice of Λ, λ and λ_N are compared at the same Reynolds number.

It follows from the definition of the relative hydraulic resistance coefficient that

$$\Lambda = \frac{\lambda}{\lambda_N} = \frac{\tau_c - \tau_c'}{\tau_N - \tau_c'} \tag{3.1}$$

We can define $\tau_N - \tau_c'$ from the condition that the corresponding average velocities are equal

$$\tfrac{1}{4}\varphi_2 R(\tau_N - \tau_c') = \tfrac{1}{4}\varphi_2 R(\tau_c - \tau_c')\,[1 + \tfrac{4}{5}\vartheta(\tau_c'' - \tau_c')\ln \Delta\tau]$$

Thus we have

$$\tau_N - \tau_c' = (\tau_c - \tau_c')\,[1 + \tfrac{4}{5}\vartheta(\tau_c'' - \tau_c')\ln \Delta\tau] \tag{3.2}$$

It follows from this equation that as the shear stress increases, $\tau_N - \tau_c$ decreases and is zero when $\tau = \tau_c''$, at which value the relative hydraulic resistance coefficient Λ becomes unity.

Substituting (3.1) in (3.2) and introducing the concept of the relative hydraulic conductivity, $\zeta = 1/\Lambda$, we obtain

$$\zeta = 1 + \tfrac{4}{5}\vartheta(\tau_c'' - \tau_c')\ln \Delta\tau \tag{3.3}$$

Thus it follows that for a structured fluid there is a relation, similar to (1.1),

$$\Delta\zeta = \ln \Delta\tau$$

between the relative hydraulic conductivity

$$\Delta\zeta = \frac{1}{0.8\vartheta}\,\frac{\zeta - 1}{\tau_c'' - \tau_c'}$$

and the nondimensional stress $\Delta\tau$.

The relative hydraulic conductivity coefficient ζ makes it possible for us to define the flow parameters for a fluid with structured viscosity by calculating the corresponding quantities for Newtonian flow with constant fluidity φ_2 equal to the largest value of the fluidity for the structured medium under consideration for given pressure gradient

$$\varphi = \varphi_2\zeta, \quad \langle w \rangle = \langle v \rangle \zeta, \quad \mathrm{Re} = \mathrm{Re}_N \zeta$$

We use this equation to obtain an expression for the Reynolds number in terms of the hydraulic resistance coefficient

$$\mathrm{Re} = \mathrm{Re}_N \zeta = \tfrac{1}{2}\rho R^2 \varphi_2 \zeta \tau_c = a\zeta\tau_c$$

Substituting this in (3.3), we obtain the corresponding resistance law (for $\tau_c' \approx 0$)

$$1/\Lambda = A \ln \mathrm{Re}\,\Lambda + B \tag{3.4}$$

where

$$A = \tfrac{4}{5}\vartheta\tau_c'', \quad B = 1 - A \ln a\tau_c''$$

Experimental results on the motion of blood in circular cylindrical channels are compared with Eq. (3.4) in Fig. 7. The discontinuity in the straight line corresponds to the establishment of fluid flow conditions at constant fluidity.

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